Modal analysis of Converter Based Power Systems

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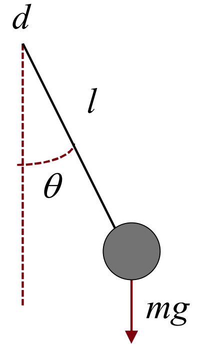
Agenda

- Stability of Nonlinear Systems: Fundamentals
- LTI Modelling of converter and grid systems
- Modal decomposition
- Practical Example: HVDC Connected Offshore Wind Farm

Example: Pendulum

Pendulum



$$\underbrace{ml\ddot{\theta}}_{\text{angular acceleration}} = \underbrace{-mg\sin(\theta)}_{\text{gravity}} - \underbrace{kl\dot{\theta}}_{\text{friction}}$$

Assign:

$$\omega = \dot{\theta}$$

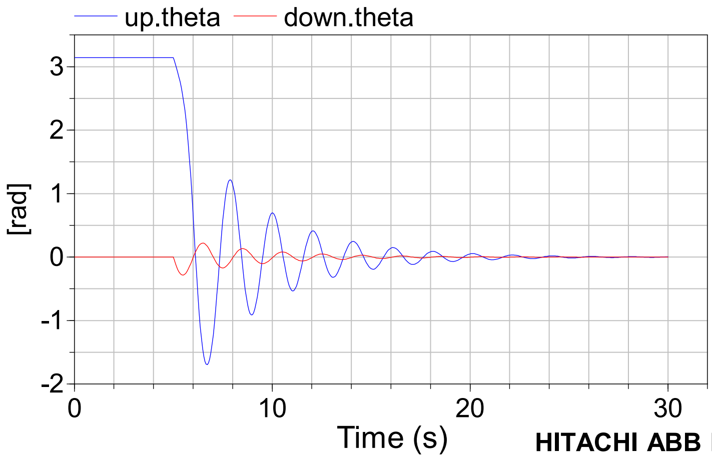
$$k = 1, l = 1, m = 1, g = 9.82$$

Non-linear state space form

$$\dot{x} = f(x)$$

with

$$f(x) = \begin{bmatrix} \omega \\ -\frac{g}{l}\sin(\theta) - \frac{k}{m}\omega \end{bmatrix}$$



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Equilibrium Points

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A point $x = x^*$ in the state space is said to be an equilibrium point of the nonlinear system

$$\dot{x} = f(x)$$

if $x(t_0) = x^* \Rightarrow x(t) = x^*$, for all $t > t_0$ that is, if the state starts at $x = x^*$, it will remain at x^* for all future time.

Consequently, equilibrium points can be found by solving the equation

$$f(x) = 0$$

See [1]

Pendulum Example:

$$f(x) = \begin{bmatrix} \omega \\ -\frac{g}{l}\sin(\theta) - \frac{k}{m}\omega \end{bmatrix} = 0 \Rightarrow \omega = 0, \theta = n\pi \text{ for all integer } n$$

Stability of Equlibrium Points

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If $x = x^*$ is an equilibrium point of the nonlinear system where $f : D \to R^n$ is continuously differentiable and D is a neighbourhood of $x = x^*$. Let

$$A = \frac{\partial f}{\partial x}(x)\bigg|_{x=x^*}$$

Then,

1. The equilibrium point $x = x^*$ is asymptotically stable if $Re\,\lambda_i < 0$ for all eigenvalues of A.
2. The equilibrium point $x = x^*$ is unstable if $Re\,\lambda_i > 0$ for one or more of the eigenvalues of A.

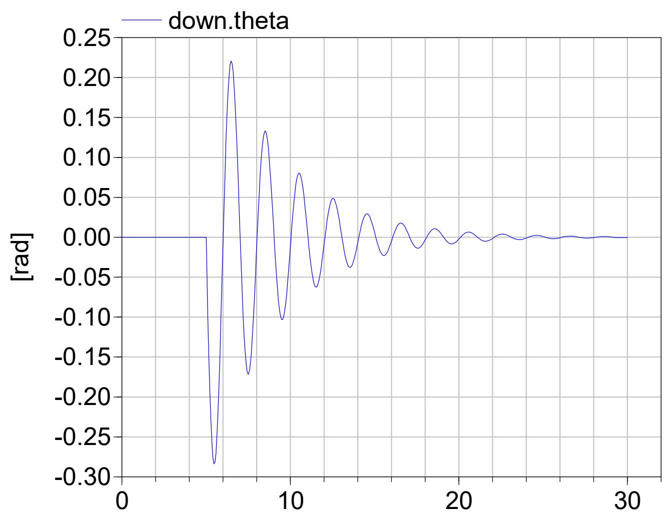
See [1]

$$A = \frac{\partial f}{\partial x} = \begin{bmatrix} 0 & 1 \\ -\frac{g}{l}\cos(\theta) & -\frac{k}{m} \end{bmatrix}$$

$$\begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix} = \begin{pmatrix} -\dfrac{k - \sqrt{\dfrac{k^2 l - 4 g m^2 \cos(x_1)}{l}}}{2m} \\ -\dfrac{k + \sqrt{\dfrac{k^2 l - 4 g m^2 \cos(x_1)}{l}}}{2m} \end{pmatrix}'$$

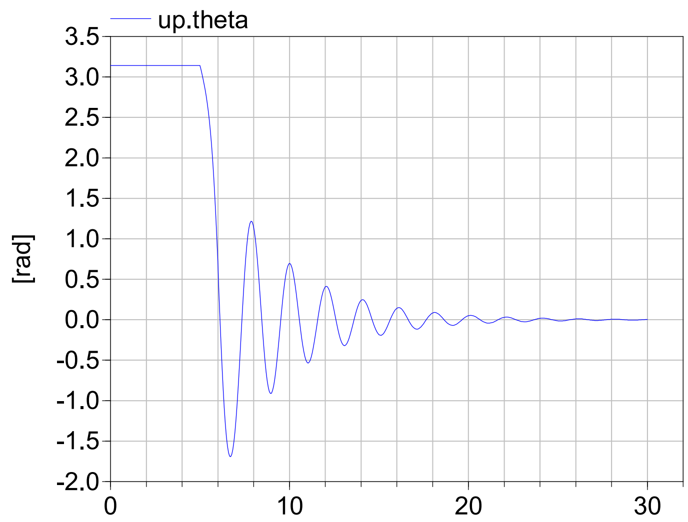
Pendulum down - Stable equilibrium

$$x = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow \lambda_{1,2} = \begin{bmatrix} -0.2500 - j3.1237 \\ -0.2500 + j3.1237 \end{bmatrix}$$



Pendulum upright - Unstable equilibrium

$$x = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow \lambda_{1,2} = \begin{bmatrix} 2.8936 \\ -3.3936 \end{bmatrix}$$



Conditions

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Eigenvalues of Jacobian matrix (A) are effective indicators of stability of nonlinear systems

But, observe following conditions:

- Equilibrium points need to exist => LTI modelling
- Continuously differentiable f(x) => Approximation of converter switching and sampling behaviour
- Jacobian matrix (A) needs to be calculated => Computer-aided modelling and automatic linearization
- Validity only in a limited neighbourhood (D) of equilibrium point => robust analysis

A Typical Converter System

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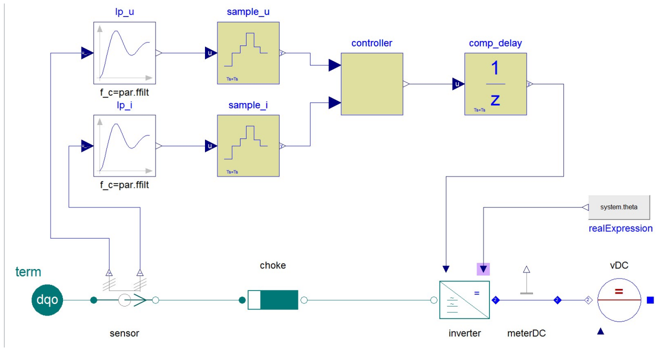
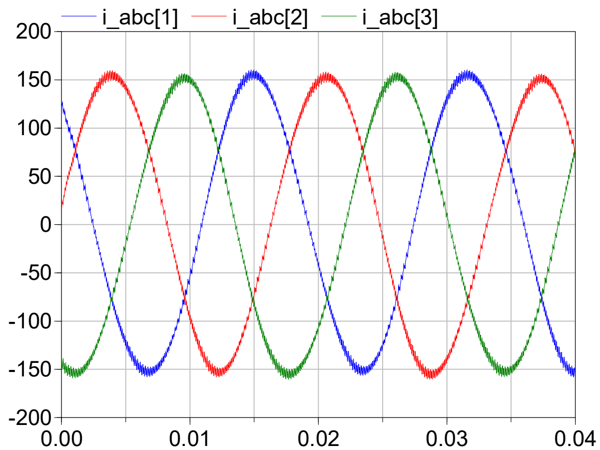
Two-level Converter with Control





Observations

Violates assumptions for eigenvalue based analysis

- Switching harmonics
- Time periodic signals
- Sampling in control system

To do:

- Transform converter from switched to average model
- Transform sampled elements to continuous time
- Transform continuous LTP elements to LTI form

Coordinate Changes
abc, dq and alpha-beta frames

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Transformation Matrices

Destination frame (columns) / Origin frame (rows)

T	$d_{abc}=\begin{bmatrix} d_a \\ d_b \\ d_c \end{bmatrix}$	$d_{\alpha\beta 0}=\begin{bmatrix} d_\alpha \\ d_\beta \\ d_0 \end{bmatrix}$	$o_{dq0}=\begin{bmatrix} o_d \\ o_q \\ o_0 \end{bmatrix}$
$o_{abc}=\begin{bmatrix} o_a \\ o_b \\ o_c \end{bmatrix}$	I	C	P
$o_{\alpha\beta 0}=\begin{bmatrix} o_\alpha \\ o_\beta \\ o_0 \end{bmatrix}$	C^T	I	R^T
$o_{dq0}=\begin{bmatrix} o_d \\ o_q \\ o_0 \end{bmatrix}$	P^T	R	I

$$\text{examples}: \mathrm{v}_{dq0} = C v_{ab0},\ \mathrm{v}_{abc} = P^T v_{dq0}$$

Look up formulas

Clarke Transform

$$C = \sqrt{\frac{2}{3}}\begin{pmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{pmatrix},\ C^T = C^{-1}$$

Rotation in dq0-frame

$$R = \begin{pmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{pmatrix},\ R(-\theta) = R^T$$

Park transform

$$P = R^T C,\ \text{with } \theta = 2\pi f_{nom} t$$
$$P^T = P^{-1}$$

Park transform derivative

$$\frac{dP}{dt} = \omega \mathrm{J}_{dq0}^T P \qquad \mathrm{J}_{dq0} = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$
$$\frac{dP^T}{dt} = \omega P^T \mathrm{J}_{dq0}$$

LTP vs LTI modelling of Three phase AC Power systems

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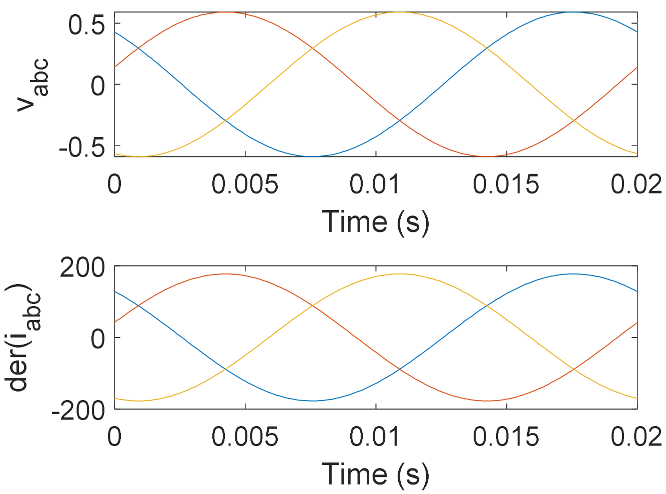
Inductor in abc frame

Stationary conditions do not correspond to an equilibrium point:

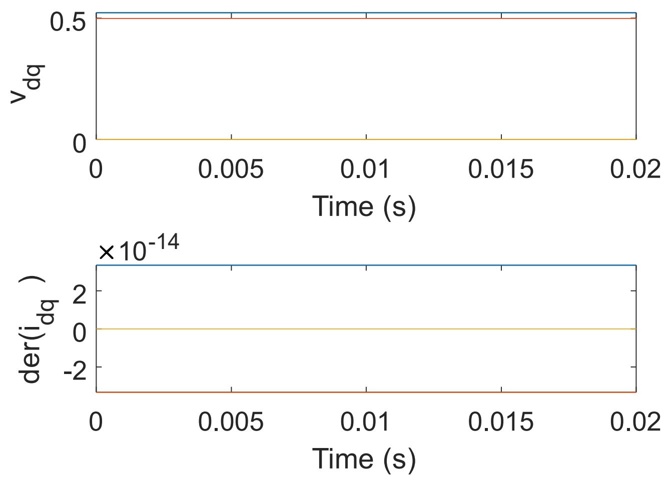
$$L_{abc}\frac{di_{abc}}{dt} = v_{abc} \qquad L_{abc} = \begin{bmatrix} L_s & L_m & L_m \\ L_m & L_s & L_m \\ L_m & L_m & L_s \end{bmatrix}$$



Inductor in dq frame

Stationary conditions correspond to equilibrium points

$$L_{dq0}\frac{di_{dq0}}{dt} + J_{dq0}\omega_s i_{dq0} = v_{dq0} \qquad L_{dq0} = \begin{bmatrix} L_s - L_m & 0 & 0 \\ 0 & L_s - L_m & 0 \\ 0 & 0 & L_s + 2L_m \end{bmatrix}$$



Derivation

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Inductor equations

$$L_{abc}\frac{d(i_{abc})}{dt}=u_{abc}$$

$$L_{abc}\frac{d(P^T i_{dqo})}{dt}=P^T u_{dqo}$$

$$L_{abc}\left(P^T\frac{di_{dqo}}{dt}+\frac{dP^T}{dt}i_{dqo}\right)=P^T u_{dqo}$$

$$L_{abc}\left(P^T\frac{di_{dqo}}{dt}+\omega P^T \mathrm{J}_{dq0} i_{dqo}\right)=P^T u_{dqo}$$

$$PL_{abc}\left(P^T\frac{di_{dqo}}{\partial t}+\omega P^T \mathrm{J}_{dq0} i_{dqo}\right)=PP^T u_{dqo}$$

$$PL_{abc}P^T\frac{di_{dqo}}{dt}+PL_{abc}\omega P^T \mathrm{J}_{dq0} i_{dqo}=PP^T u_{dqo}$$

$$L_{dq0}\left(\frac{di_{dqo}}{dt}+\omega \mathrm{J}_{dq0} i_{dqo}\right)=u_{dqo}$$

Impedance Matrix Transformation

Symmetrical case

$$L_{abc}=\begin{bmatrix} L_s & L_m & L_m \\ L_m & L_s & L_m \\ L_m & L_m & L_s \end{bmatrix}$$

$$L_{dq0}=PL_{abc}P^T=\begin{bmatrix} L_s-L_m & 0 & 0 \\ 0 & L_s-L_m & 0 \\ 0 & 0 & L_s+2L_m \end{bmatrix}$$

Results in time varying impedance matrices in the unsymmetrical case

Converter Average Approximation

- Two and three level converters can be represented by a gain and a AC/DC power balance equation
- Assumes a constant gain for all frequencies
- Accuracy mainly dependent on carrier frequency/pulse number
- Overmodulation and multilevel converters require special treatment

- Different models for different modulation strategies
- See e.g. [3] for details

Accuracy of Average Model

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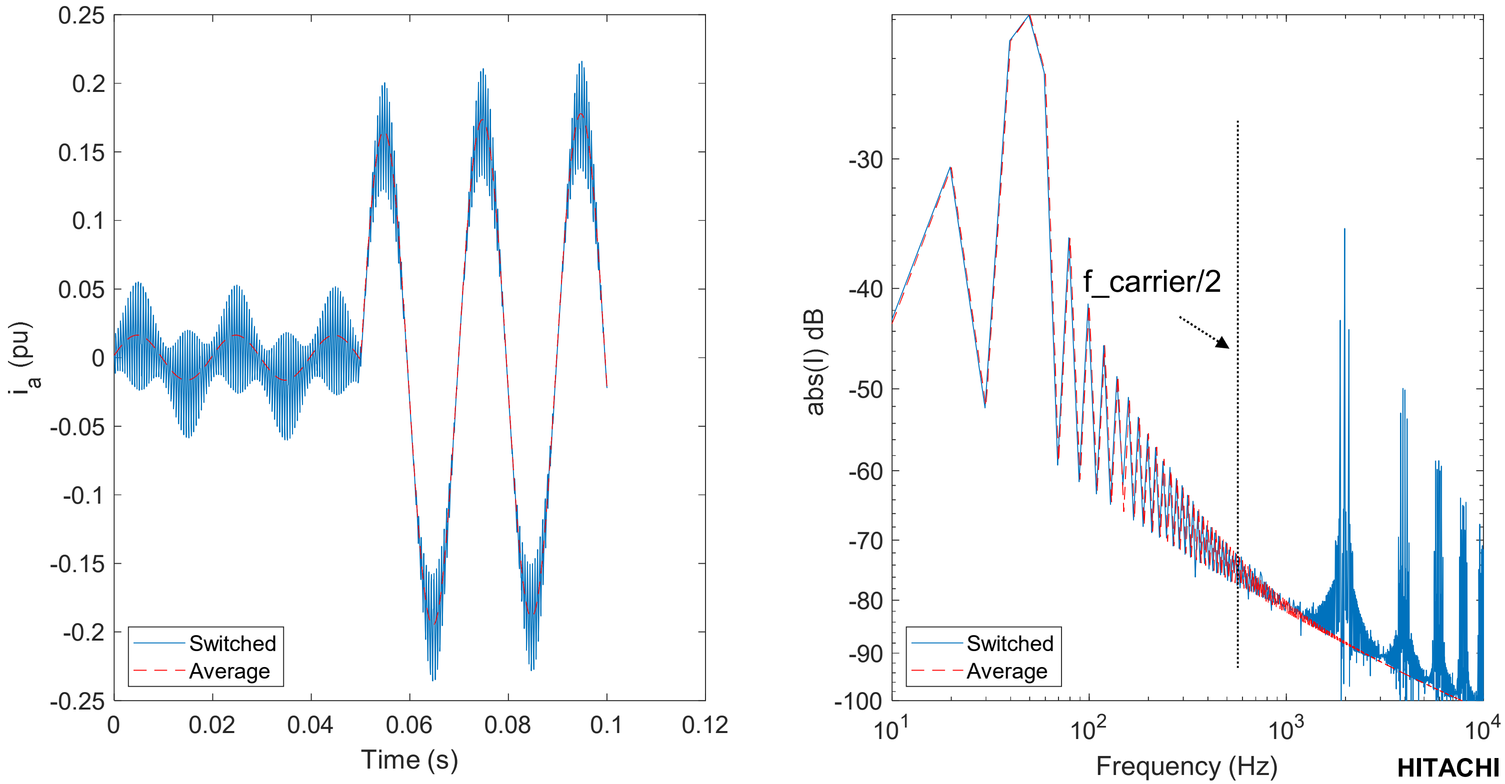
- Example: sinusoidal PWM with 2kHz carrier, step in modulation index magnitude at 0.05 s



Sampling Approximation

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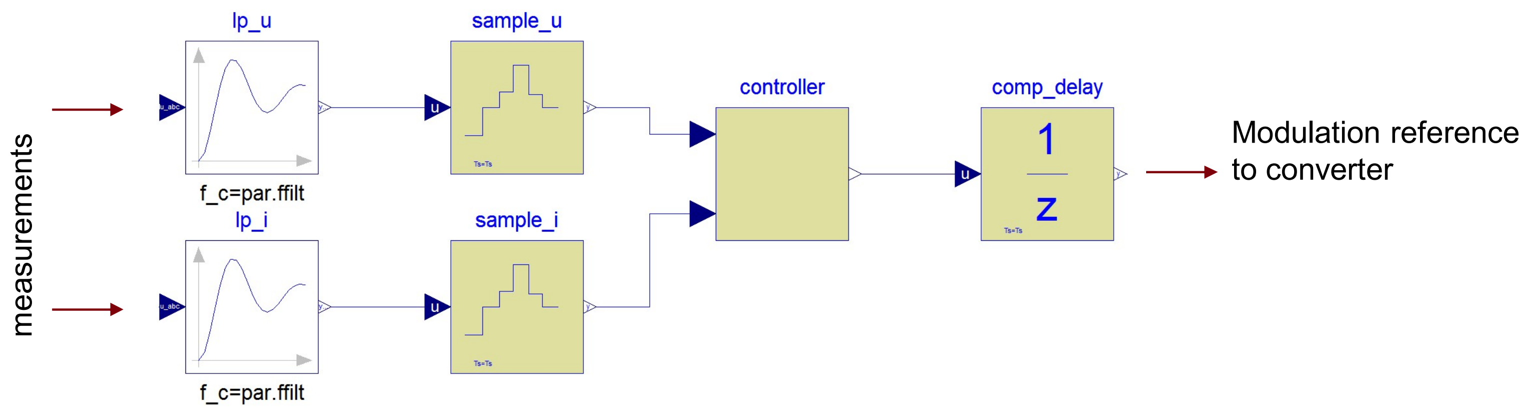

- Pade approximation of sampling delay
- Pade approximation of computation delay
- Transforming controllers to continuous time equivalents

Sampling Approximation

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- Zero order hold

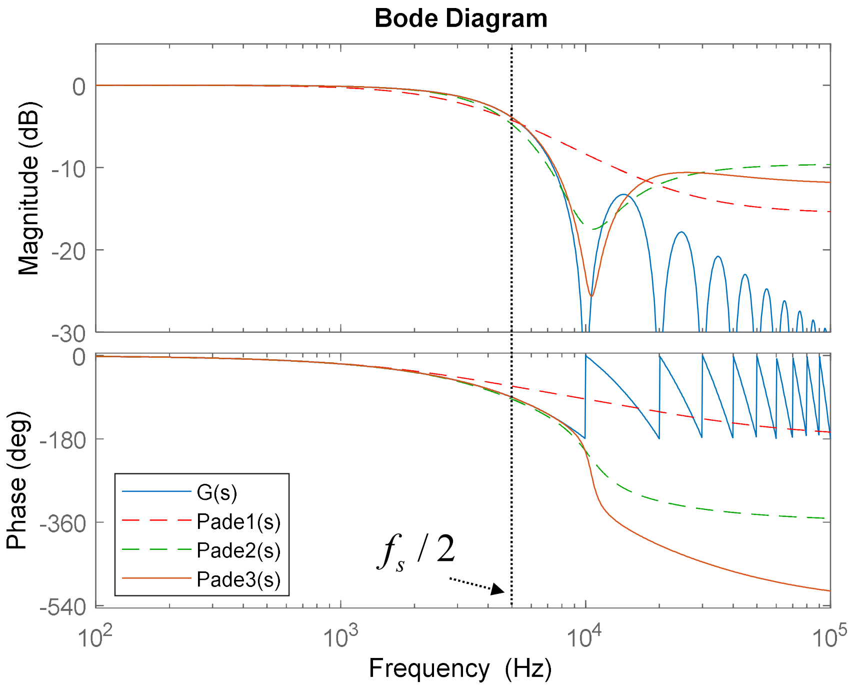
$$G(s) = \frac{1 - e^{-sT}}{sT}$$

- However not well suited for linearization
- Instead use Pade-approximation of

$$Pade_1(s) = \frac{-60Ts + 840}{360Ts + 840}$$

$$Pade_2(s) = \frac{20(Ts)^2 - 60Ts + 840}{60(Ts)^2 + 360Ts + 840}$$

$$Pade_3(s) = \frac{-(Ts)^3 + 20(Ts)^2 - 60Ts + 840}{(4Ts)^3 + 60(Ts)^2 + 360Ts + 840}$$



Time Delay Approximation

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Time delay can be represented by a transfer function

$$G(s) = \mathrm{e}^{-sT}$$

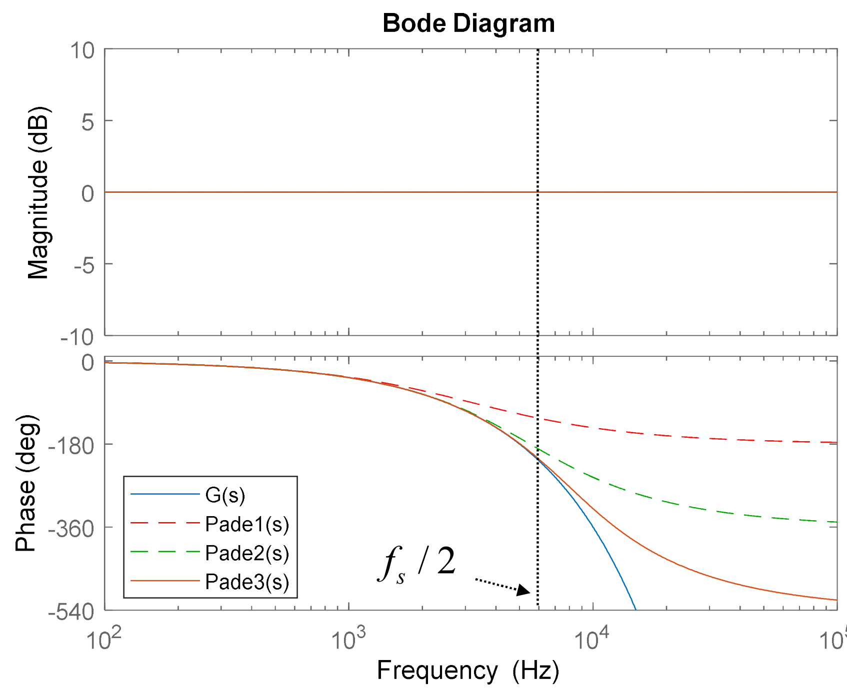
However not well suited for linearization

Pade-approximation

$$Pade_1(s) = \frac{-60(Ts)+120)}{+60(Ts)+120)}$$

$$Pade_2(s) = \frac{12(Ts)^2 - 60(Ts)+120)}{12(Ts)^2 + 60(Ts)+120)}$$

$$Pade_3(s) = \frac{-(Ts)^3 + 12(Ts)^2 - 60(Ts)+120)}{(Ts)^3 + 12(Ts)^2 + 60(Ts)+120)}$$



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Approximation of discrete time systems

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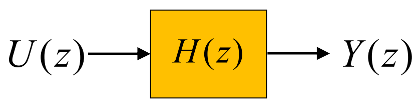
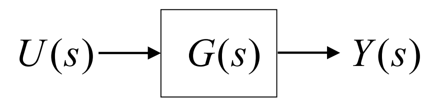
Discrete Time

Example: PI Controller



$$H(z)=\frac{\left(k_p\left(2T_i+T_s\right)\right)z-k_p\left(2T_i-T_s\right)}{\left(2T_i\right)z-2T_i}$$

Continuous Time



$$s=\frac{2\left(z-1\right)}{T_s\left(z+1\right)}$$

$$\longleftrightarrow$$

$$z=-\frac{(T_s s+2)}{(T_s s-2)}$$

$$G(s)=k(1+\frac{1}{Ts})$$

See [4] for details.

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Approximation Accuracy

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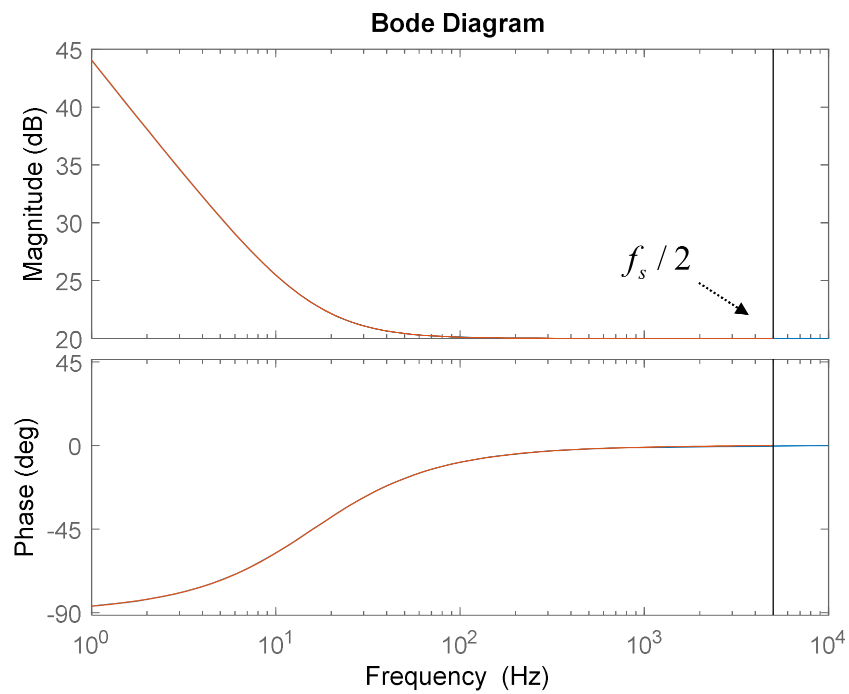
PI Controller

$$k_p = 10, T_i = 0.01, T_s = 0.0001s$$

$$G(s) = \frac{0.1\ s + 10}{0.01\ s}$$

$$H(z) = \frac{0.201\ z - 0.199}{0.02\ z - 0.02}$$



LTI Conversion of LTP Control Blocks

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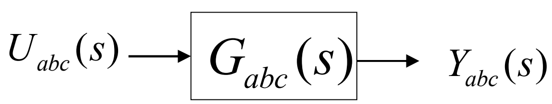
abc Frame

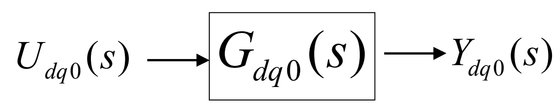


$$sX_{abc} = A_{abc}X_{abc} + B_{abc}U_{abc}$$
$$Y_{abc} = C_{abc}X_{abc} + D_{abc}U_{abc}$$

$$s \rightarrow sI + \omega_s J_{dq0}$$

$$A_{dq0} = A_{abc} - \omega_s J_{dqo}$$
$$B_{dq0} = B_{abc}$$
$$C_{dq0} = C_{abc}$$
$$D_{dq0} = D_{abc}$$

dq Frame



$$sX_{dq0} = A_{dq0}X_{dq0} + B_{dq0}U_{dq0}$$
$$Y_{dq0} = C_{dq0}X_{dq0} + D_{dq0}U_{dq0}$$

$$G_{dq0}(s) = C_{dq0}(sI - A_{dq0})^{-1}B_{dq0} + D_{u_{dq0}}$$

Example: Low Pass filter

Low pass filter

$$G_{abc}(s)=\begin{pmatrix}\frac{k}{Ts+1} & 0 & 0\\ 0 & \frac{k}{Ts+1} & 0\\ 0 & 0 & \frac{k}{Ts+1}\end{pmatrix}$$

$$k=1, T=1/(2\pi 500), \omega_s=2\pi 50$$

$$sX_{abc}=\begin{pmatrix}-400\pi & 0 & 0\\ 0 & -400\pi & 0\\ 0 & 0 & -400\pi\end{pmatrix}X_{abc}+\begin{pmatrix}32 & 0 & 0\\ 0 & 32 & 0\\ 0 & 0 & 32\end{pmatrix}U_{abc}$$

$$Y_{abc}=\begin{pmatrix}\frac{25\pi}{2} & 0 & 0\\ 0 & \frac{25\pi}{2} & 0\\ 0 & 0 & \frac{25\pi}{2}\end{pmatrix}' X_{abc}+\begin{pmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{pmatrix}U_{abc}$$

$$sX_{dq0}=\begin{pmatrix}-400\pi & 100\pi & 0\\ -100\pi & -400\pi & 0\\ 0 & 0 & -400\pi\end{pmatrix}' X_{dq0}+\begin{pmatrix}32 & 0 & 0\\ 0 & 32 & 0\\ 0 & 0 & 32\end{pmatrix}U_{dq0}$$

$$Y_{dq0}=\begin{pmatrix}\frac{25\pi}{2} & 0 & 0\\ 0 & \frac{25\pi}{2} & 0\\ 0 & 0 & \frac{25\pi}{2}\end{pmatrix}' X_{dq0}+\begin{pmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{pmatrix}U_{dq0}$$

Modal Analysis - Workflow and Goals

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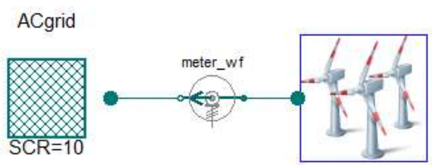
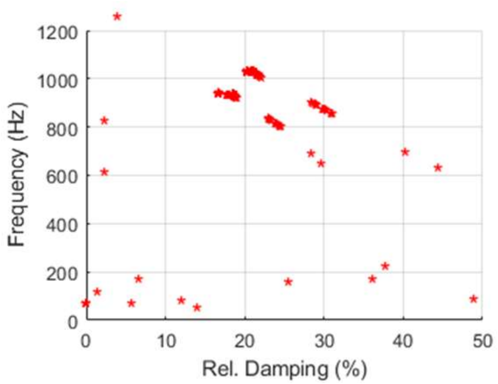
Workflow



$$\dot{x} = f(x,u,t)$$
$$y = g(x,u,t)$$

$$\Delta\dot{x} = A\Delta x + B\Delta u$$
$$\Delta y = C\Delta x + D\Delta u$$

Modal decomposition



Goals

1. Modelling of grid-converter systems

2. Linearization

3. Modal decomposition

- Stability analysis
- Root-cause analysis
- Control design studies

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Example: Inverter to Grid System

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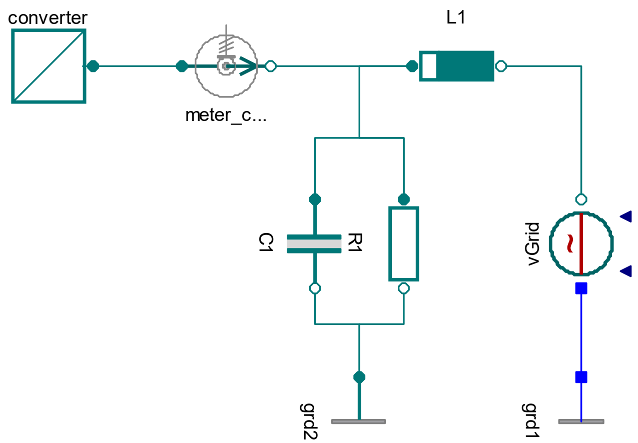
Inverter to Grid System



Components

Converter

- Converter with current control similar to those used in solar PV/wind power plant
- PLL
- Grid filter

Grid and Load

- Weak grid
- Resistive/Capacitive load

Converter

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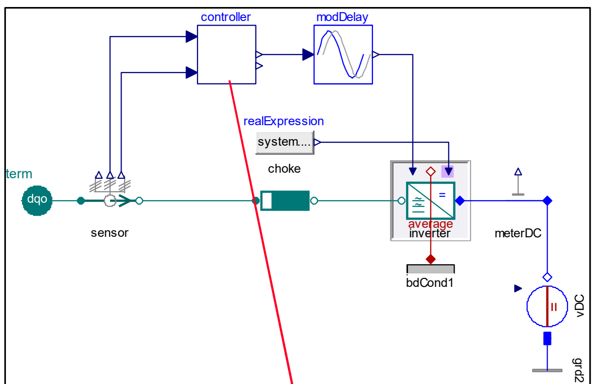
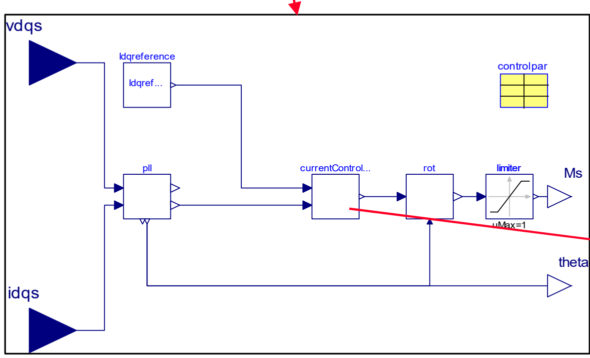
Modelling with ConverterStab

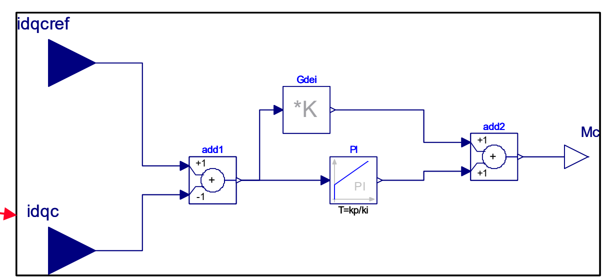




Converter

- Average model of converter
- Switching delay approximated by first order Pade approximation (0.75-1.5*Tswitch)
- PI Current control with decoupling in dq frame
- PLL with PI controller

Eliminates need to manually derive analytical expressions for input/output impedance



Simulation: Base Case

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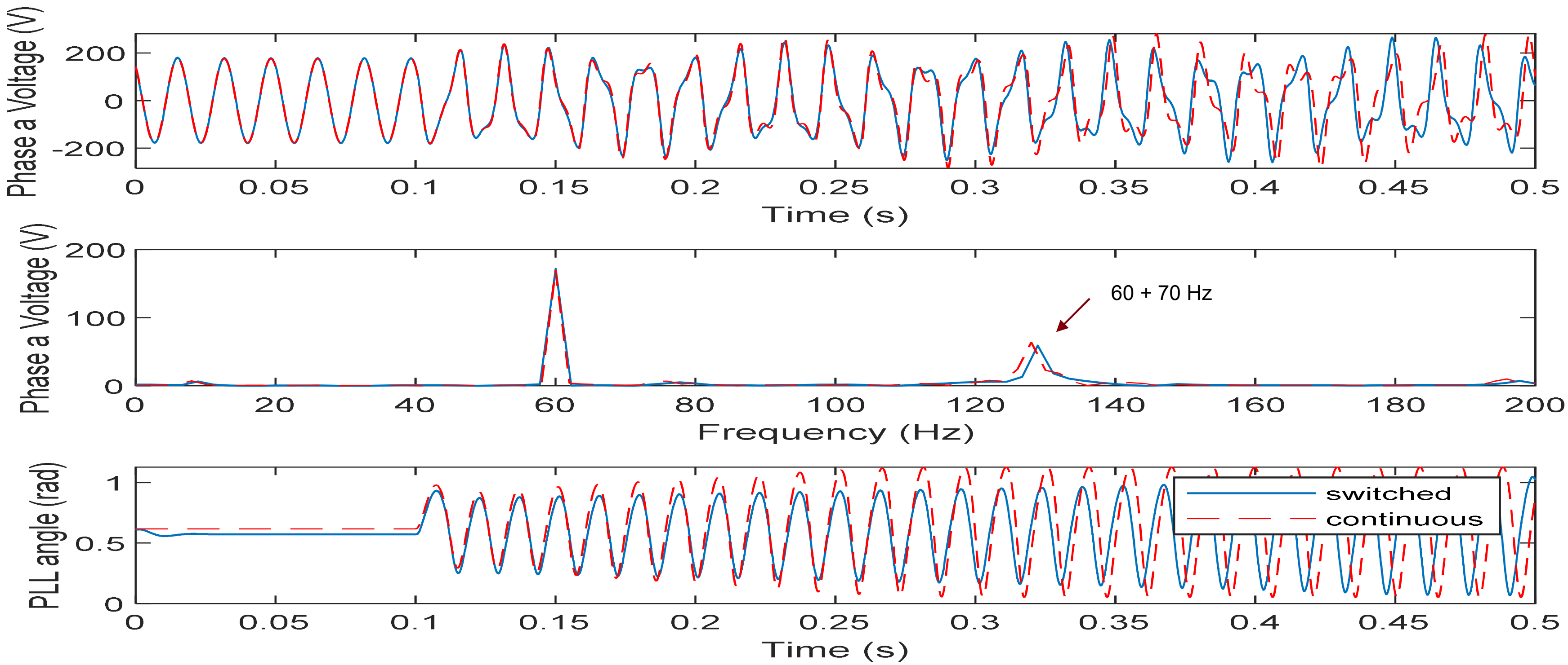

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A – System Matrix

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System Matrix

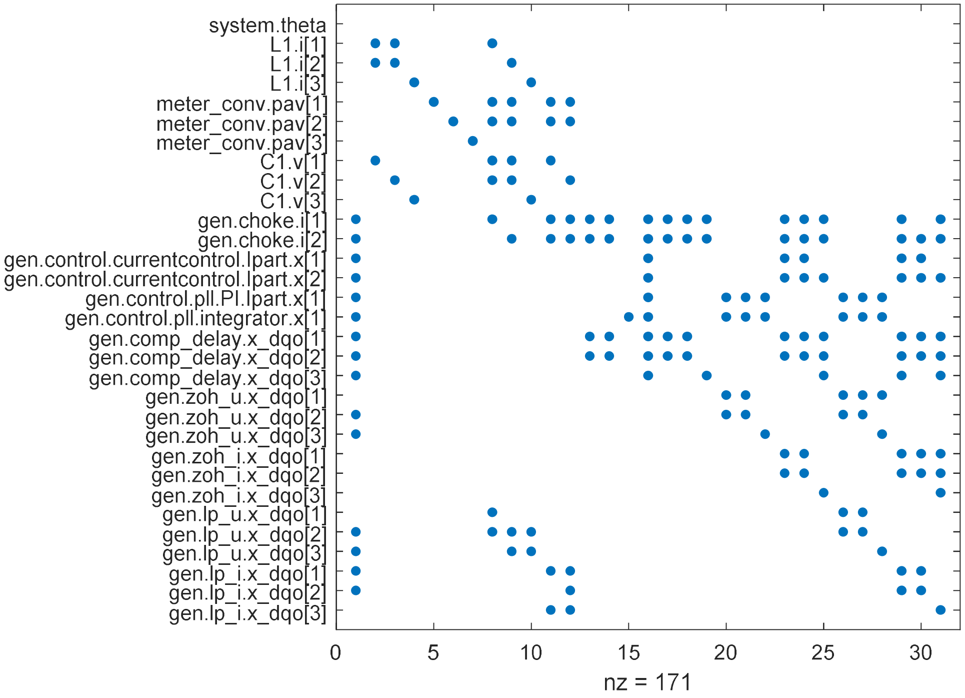
- Obtained through automatic differentiation of Modelica model
- A is a sparse matrix illustrating coupling between elements
- Eigenvalues can be used to determine stability

- The A matrix models the system response around the equilibrium point used for linearization

$$\dot{x} = f(x)$$

$$\dot{x} \approx \left.\frac{\partial f}{\partial x}\right|_{x=x^*} = Ax$$

Example



Modal Decomposition

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Given an initial state the eigenresponse of the system $\dot{\mathbf{x}} = \mathbf{A}\mathbf{x}$ is :

$$x(t) = e^{At} x(0)$$

$$e^{At} = \sum_{i=1}^{n} e^{\lambda_i t} r_i l_i \quad \text{where } \lambda_i = \alpha_i + j\omega_i \text{ are the eigenvalues of A}$$

$$\mathrm{R}=[\mathrm{r}_1, \ldots, r_n], \mathrm{L}=[\mathrm{l}_1, \ldots, l_n]^T = R^{-1}$$

$$x(t) = \sum_{i=1}^{n} e^{(\alpha_i + j\omega_i)t} r_i l_i\, x(0) = \sum_{i=1}^{n} e^{\alpha_i t} (\cos \omega_i t + j \sin \omega_i t) r_i l_i\, x(0)$$

Modal Decomposition Animation

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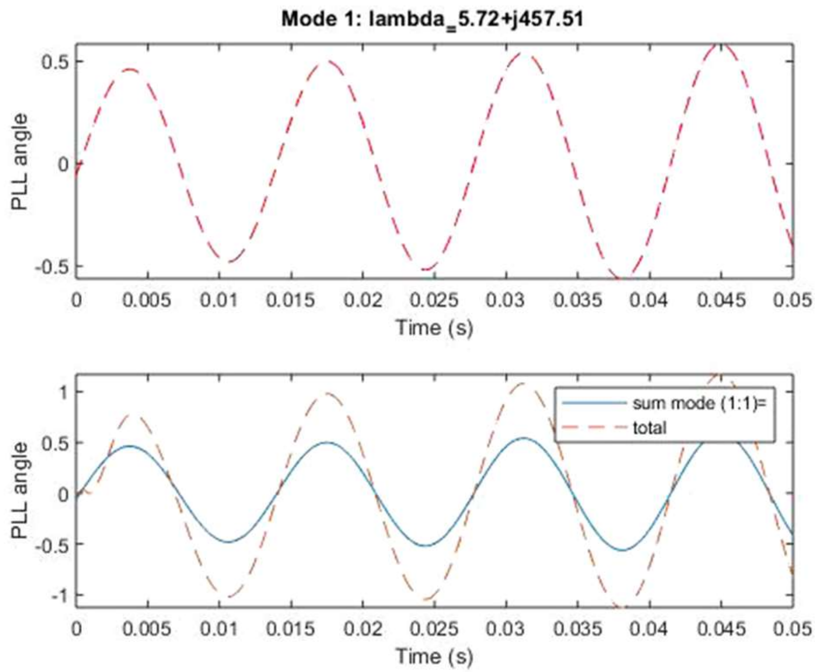
- Summing the modal contributions up results in original response



Contribution of mode n

Sum of contributions of mode 1 thru n

Mode Damping and Frequency

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System response can be written as:

$$x(t) = \sum_{i=1}^{n} \underbrace{e^{\alpha_i t}}_{\text{exponential part}} \underbrace{(\cos\omega_i t + j\sin\omega_i t)}_{\text{oscillatory part}} r_i l_i^T x(0)$$

- Right half plane eigenvalues – exponential growth
- Left half plane eigenvalues – exponential decay
- Complex eigenvalues – oscillatory response
- Real eigenvalues – non-oscillatory response

Definitions:

- Attenuation α_i
- Mode frequency ω_i
- Damping ratio $\zeta_i = \frac{-\alpha_i}{\sqrt{\alpha_i^2 + \omega_i^2}}$

Participation Factors

$$x(t) = \sum_{i=1}^{n} e^{\alpha_i t}(\cos \omega t + j \sin \omega t) \underbrace{r_i l_i}_{participation} x(0)$$

modes →

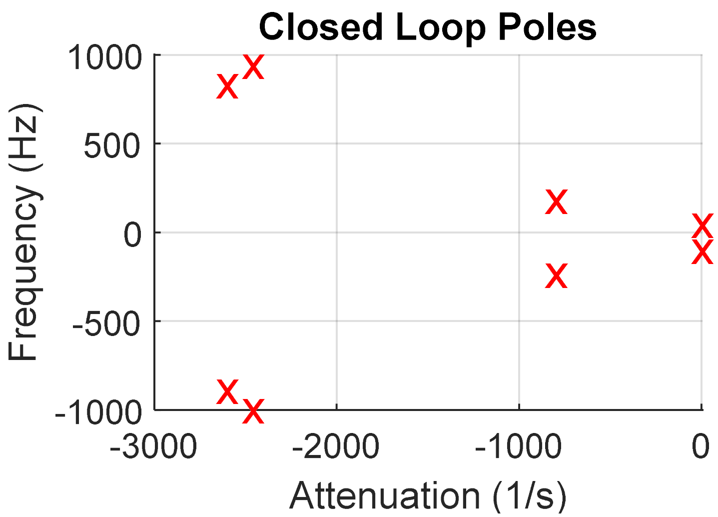
$$R \odot L^H = [r_1 \ldots r_n] \odot [l_1 \ldots l_n] = \begin{bmatrix} r_{11} l_{11} & \cdots & r_{1n} l_{1n} \\ \vdots & \ddots & \vdots \\ r_{n1} l_{n1} & \cdots & r_{nn} l_{nn} \end{bmatrix} = \begin{bmatrix} p_{11} & \cdots & p_{1n} \\ \vdots & \ddots & \vdots \\ p_{n1} & \cdots & p_{nn} \end{bmatrix}$$

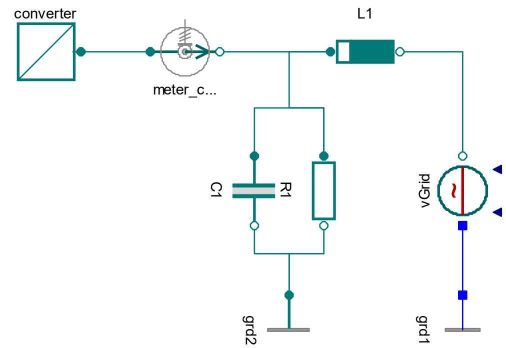
states ↓

p_{ij} is called participation factor

for state i in mode j

Modal Analysis

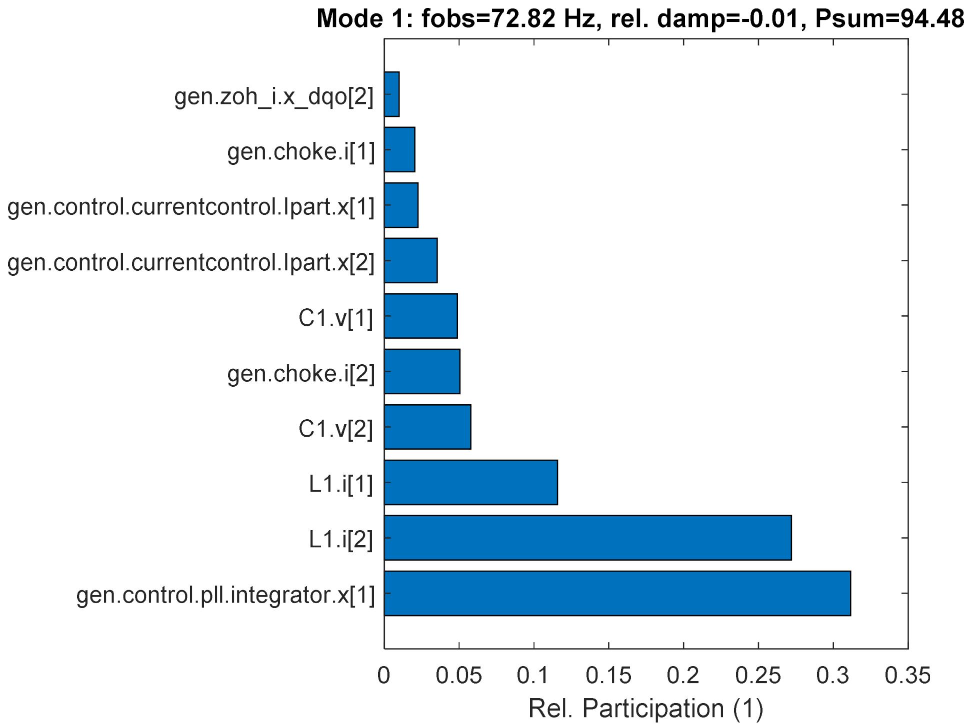




Mode #	Frequency	Damping	eq. PM	Dominant State:
1	72.83 Hz	-1.29%	-1.47	converter.controller.pll.integrator.x[1]
2	969.92 Hz	37.39%	37.22	converter.choke.i[1]
3	858.84 Hz	43.39%	41.72	converter.choke.i[1]
4	207.74 Hz	52.06%	47.64	L1.i[2]

Participation Factors for 73 Hz Mode

Participation Factors



Observations

- The PLL angle state exhibits the strongest participation in the instability -> instability is related to the PLL tuning
- The grid inductance states exhibit the second strongest participation in the instability -> connected to grid strength
- The load capacitance states exhibit the second strongest participation in the instability -> connected to the the prescence of capacitive load

Modal Sensitivity Analysis

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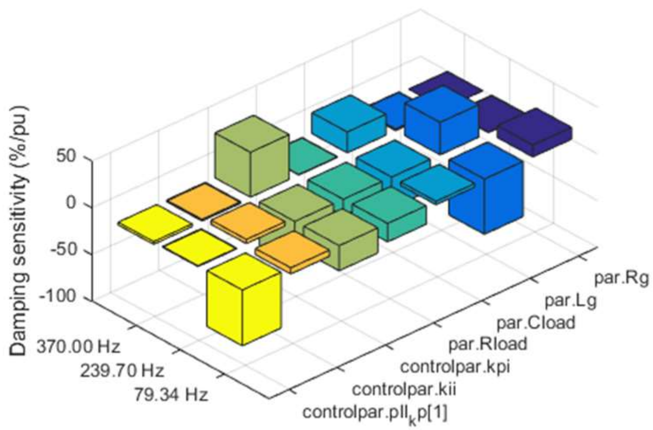
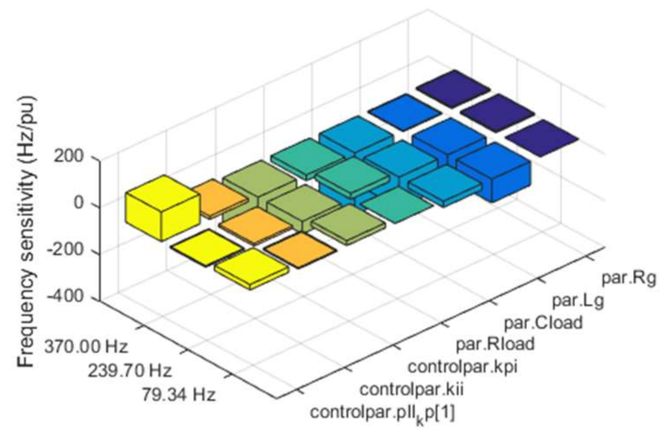
Relative Parameter Sensitivites



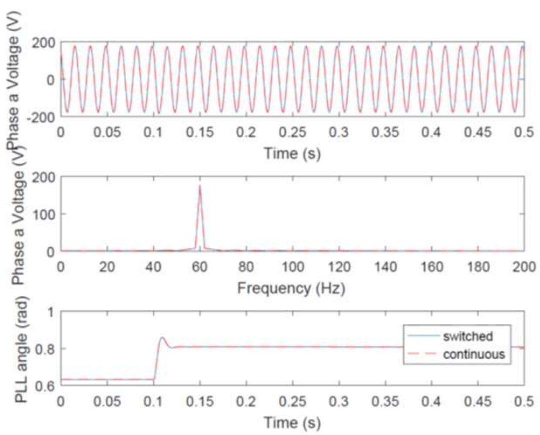


Unstable 79 Hz mode exhibits strong parameter sensitivities with respect to:

- Pll_kp – PLL proportional gain
- Lg – Grid inductance
- Rload, Cload – load resistance and capacitance

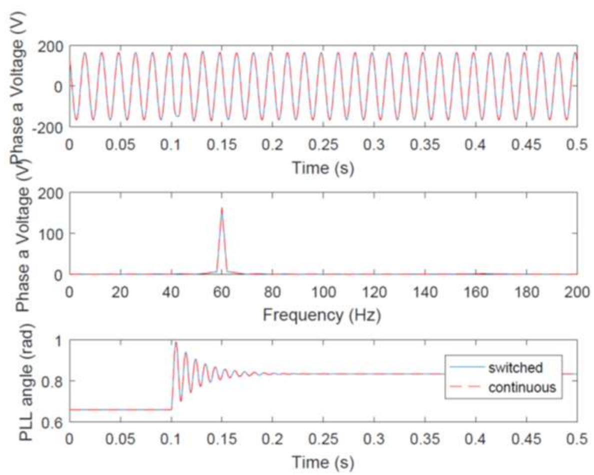
Mitigation Methods

Reduced PLL Gain



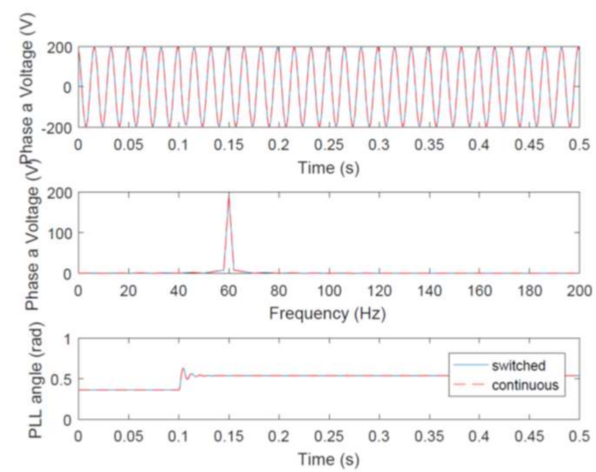
Halved PLL Gain

Reduced Capacitive Load



Capacitance 5 times smaller

Reduced Grid Impedance



Halved grid inductance

Various Modelling Workflows

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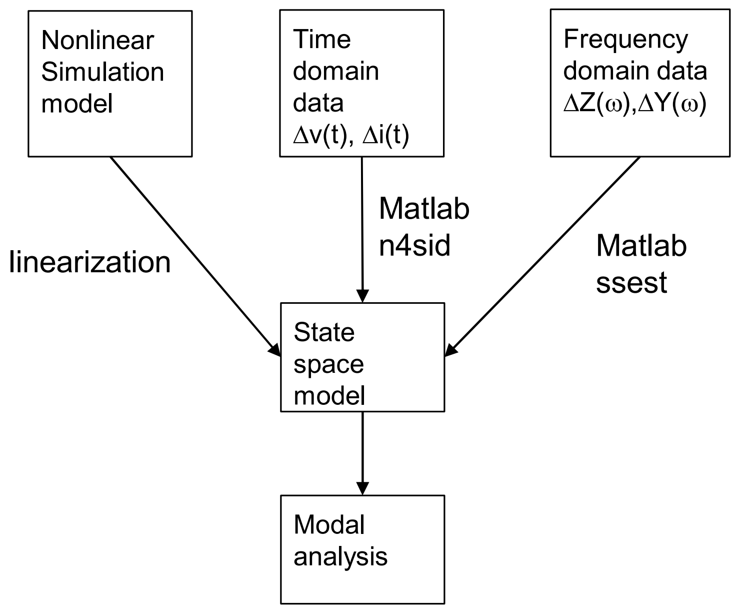

Modal analysis relies on linearized models of each component

- Transfer function can be identified from time or frequency domain data
- Modal analysis does *not require* exchange of EMT simulation models

Impedance Stability with Black Box Models

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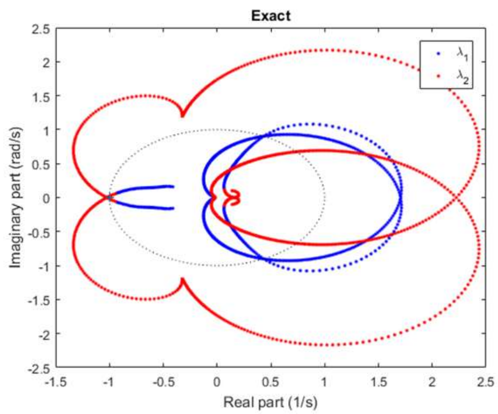
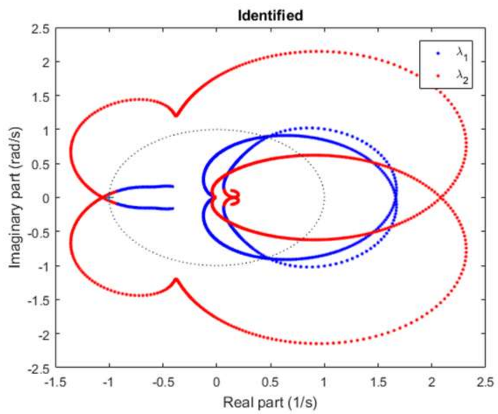
Analytic vs Estimated Converter Impedances

Exact Models



Gain Margin: -0.11 dB

Identified Models



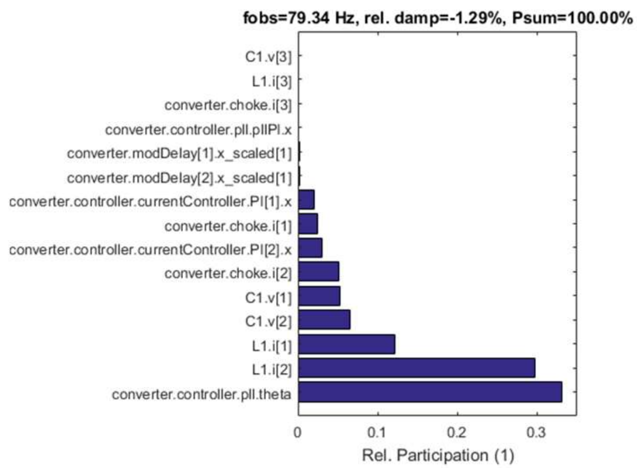
Gain Margin: -0.2 dB

Modal Analysis with Black Box Models

Analytic vs Estimated Converter Impedances

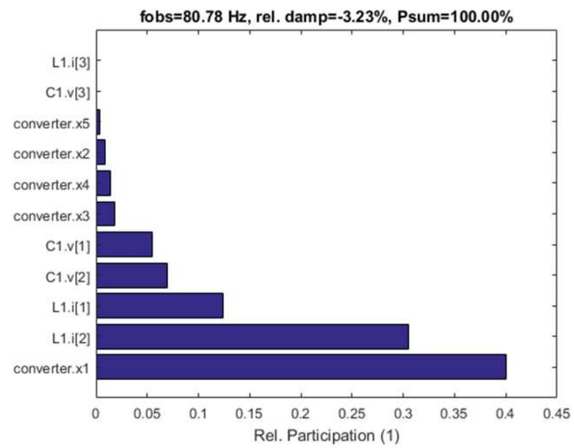
Exact Models

Mode	Frequency	Damping (%)	Dominant State:
1	79.34	-1.29%	converter.controller.pll.theta
2	239.7	53.81%	C1.v[1]
3	370	76.17%	converter.choke.i[2]



Identified Models

Mode	Frequency	Damping (%)	Dominant State:
1	80.78	-3.23%	converter.x1
2	240.51	53.49%	C1.v[1]
3	358.15	75.08%	converter.x2



Conclusion

Modal analysis offers a rigorous mathematical framework to diagnose controller stability problems and grid interactions

- Mathematical foundations go back to the days of Euler and Lagrange
- Eigenvalues and eigenvectors of the linearized system can be used to predict stability as well as identify root causes of stability problems
- Approach is practical also for large scale systems with hundreds of converters
- Care must be taken to properly approximate switching and sampling behaviour
- Good accuracy up to around half the switching frequency
- Techniques are available to integrate frequency domain models in modal analysis
- Parameter sweeps and modal sensitivity studies are useful tools to troubleshoot and optimize control design

References / Recommended Reading

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[1] Hassan K. Khalil, Nonlinear Systems, Macmillan, 1991.

[2] Modeling of Electromagnetic Transients in Power Systems". In: Modelica Workshop 2000 Proceedings. Oct. 23, 2000, pp. 93–97.

[3] Modelling and Control of Three-phase PWM Converters, Silva Hiti, PhD Thesis, Virginia Tech, 1995.

[4] KJ Aström, B. Wittenmark, Computer-Controlled Systems: Theory and Design, Third Edition, Dover Books on Electrical Engineering.